GCE AS MARKING SCHEME

SUMMER 2023

AS
FURTHER MATHEMATICS
UNIT 3 FURTHER MECHANICS A
2305U30-1

INTRODUCTION

This marking scheme was used by WJEC for the 2023 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

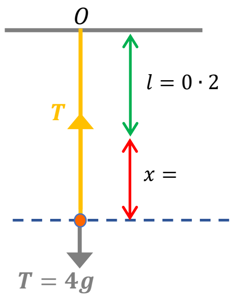
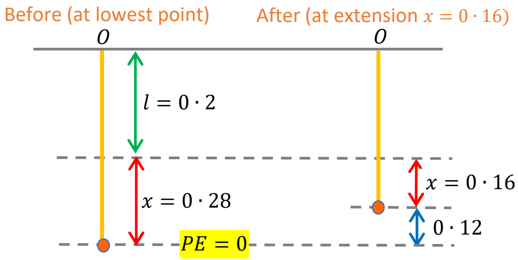
It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

WJEC GCE AS FURTHER MATHEMATICS

UNIT 3 FURTHER MECHANICS A

SUMMER 2023 MARK SCHEME

Q1	Solution	Mark	Notes
(a)	Use of Hooke's Law $T = \frac{5gx}{0 \cdot 2}$ $4g = \frac{5gx}{0 \cdot 2}$ $x = 0 \cdot 16 \quad (\text{m})$  	M1 A1 **[2]**	Used, $T = \frac{\lambda x}{l}$ $\lambda = 5g = 49 \qquad 4g = 39 \cdot 2$ Convincing
(b)	 Using expression for $\text{EE} = \frac{\lambda x^2}{2l}$ $\text{EE} = \begin{cases} \frac{5g(0 \cdot 28^2 - 0 \cdot 16^2)}{2(0 \cdot 2)} = 6 \cdot 468 = 0 \cdot 66g \\ \frac{5g(0 \cdot 28)^2}{2(0 \cdot 2)} = 9 \cdot 604 = 0 \cdot 98g \\ \frac{5g(0 \cdot 16)^2}{2(0 \cdot 2)} = 3 \cdot 136 = 0 \cdot 32g \end{cases}$ Using expression for $\text{PE} = mgh$ $\text{PE} = \begin{cases} 4g(0 \cdot 28 - 0 \cdot 16) = 4 \cdot 704 = 0 \cdot 48g \\ 4g(0 \cdot 28) = 10 \cdot 976 = 1 \cdot 12g \\ 4g(0 \cdot 16) = 6 \cdot 272 = 0 \cdot 64g \\ 4g(0 \cdot 48) = 18 \cdot 816 = 1 \cdot 92g \\ 4g(0 \cdot 36) = 14 \cdot 112 = 1 \cdot 44g \end{cases}$ $\text{KE} = \frac{1}{2}(4)v^2$ Conservation of Energy (before = after) $\frac{5g(0 \cdot 28)^2}{2(0 \cdot 2)} = \frac{1}{2}(4)v^2 + \frac{5g(0 \cdot 16)^2}{2(0 \cdot 2)} + 4g(0 \cdot 12)$ $9 \cdot 604 = 2v^2 + 3 \cdot 136 + 4 \cdot 704$ $v^2 = 0 \cdot 882 \qquad (= 0 \cdot 09g)$ $v = 0 \cdot 9(391 \ldots) \quad (\text{ms}^{-1})$	M1 A1 M1 A1 B1 M1 A1 A1 **[8]**	 A correct expression A correct expression KE, EE and PE all present All correct, oe (see further notes) Note $v = \frac{21\sqrt{5}}{50}$ or $v = 0 \cdot 3\sqrt{g}$
	Total for Question 1	**10**	

Further Notes

(b)

Alternative Solution for Conservation of Energy A1

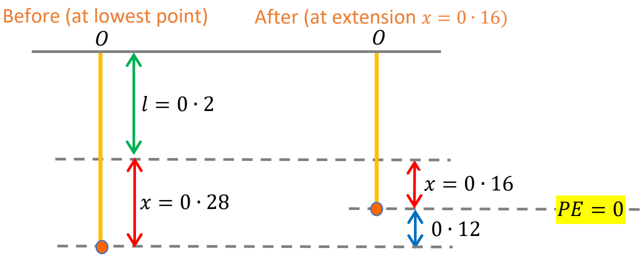
KE = loss in EE - gain in PE

$\frac{1}{2}(4)v^2 = \frac{5g(0{\cdot}28^2 - 0{\cdot}16^2)}{2(0{\cdot}2)} - 4g(0\cdot28 - 0\cdot16)$ A1

$2v^2 = 6\cdot468 - 4\cdot704$ $\left(2v^2 = \frac{33}{50}g - \frac{12}{25}g \quad \text{or} \quad 2v^2 = 0\cdot64g - 0\cdot48g\right)$

(b)

Alternative PE reference points

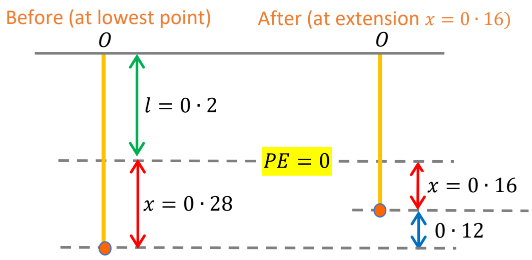


$\frac{5g(0{\cdot}28)^2}{2(0{\cdot}2)} - 4g(0\cdot12) = \frac{1}{2}(4)v^2 + \frac{5g(0{\cdot}16)^2}{2(0{\cdot}2)}$ A1

$9\cdot604 - 4\cdot704 = 2v^2 + 3\cdot136$

(b)

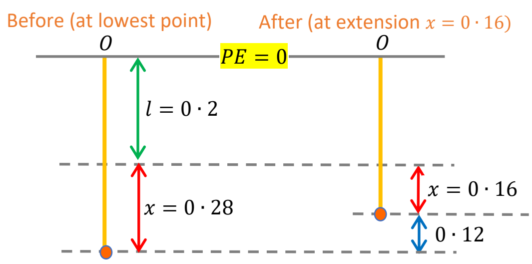
Alternative PE reference points



$\frac{5g(0{\cdot}28)^2}{2(0{\cdot}2)} - 4g(0\cdot28) = \frac{1}{2}(4)v^2 + \frac{5g(0{\cdot}16)^2}{2(0{\cdot}2)} - 4g(0\cdot16)$ A1

$9\cdot604 - 10\cdot976 = 2v^2 + 3\cdot136 - 6\cdot272$

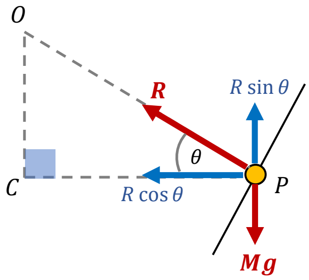
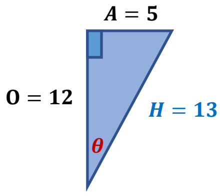
(b)

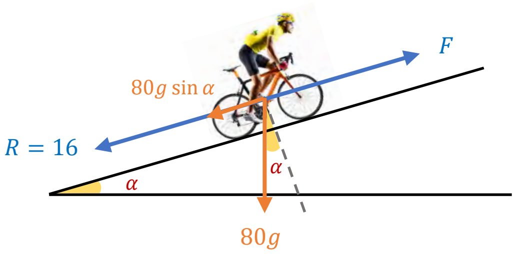


$\frac{5g(0{\cdot}28)^2}{2(0{\cdot}2)} - 4g(0\cdot48) = \frac{1}{2}(4)v^2 + \frac{5g(0{\cdot}16)^2}{2(0{\cdot}2)} - 4g(0\cdot36)$ A1

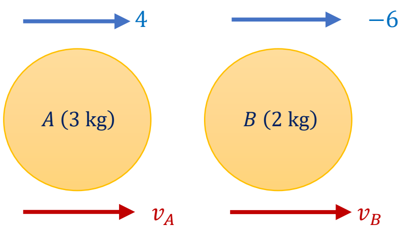
$9\cdot604 - 18\cdot816 = 2v^2 + 3\cdot136 - 14\cdot112$

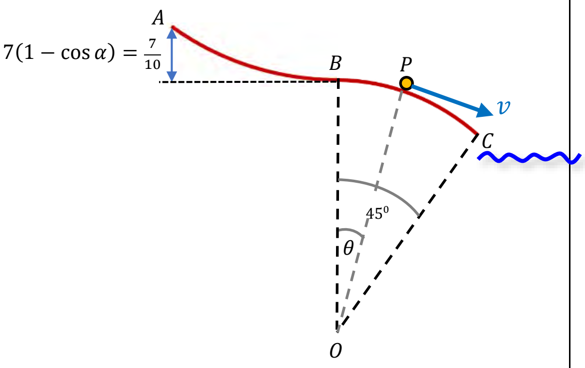
Q2	Solution	Mark	Notes
(a)	$\mathbf{r}_A = \begin{cases} 6\mathbf{i} + 21\mathbf{j} - 8\mathbf{k} + (3\mathbf{i} - \mathbf{j} + 4\mathbf{k})t \\ \begin{pmatrix} 6+3t \\ 21-t \\ 4t-8 \end{pmatrix} \\ (6+3t)\mathbf{i} + (21-t)\mathbf{j} + (4t-8)\mathbf{k} \end{cases}$	B1	
	At $t = 5$, $\mathbf{r}_A = 21\mathbf{i} + 16\mathbf{j} + 12\mathbf{k}$	M1	Use of $t = 5$
	$\lvert\mathbf{r}_A\rvert = \sqrt{21^2 + 16^2 + 12^2}$	m1	Attempt to find $\lvert\mathbf{r}_A\rvert$
	$\lvert\mathbf{r}_A\rvert = \sqrt{841} = 29 \quad (\text{m})$	A1	cao
		[4]	
(b)	(i) $\mathbf{v} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$	M1	
	$\mathbf{v}_B = \frac{3}{2}\cos\left(\frac{t}{2}\right)\mathbf{i} + \frac{3}{2}\sin\left(\frac{t}{2}\right)\mathbf{j}$	A1	
	$\lvert\mathbf{v}_B\rvert = \sqrt{\left(\frac{3}{2}\cos\left(\frac{t}{2}\right)\right)^2 + \left(\frac{3}{2}\sin\left(\frac{t}{2}\right)\right)^2}$	m1	FT their $\mathbf{v}_B$ throughout
	$\lvert\mathbf{v}_B\rvert = \sqrt{\frac{9}{4}} = \frac{3}{2} = 1\cdot5 \quad (\text{ms}^{-1})$ which is constant.	A1	cao
	(ii) Dot product, $\mathbf{v}_A . \mathbf{v}_B = 0$	M1	
	$(3\mathbf{i} - \mathbf{j} + 4\mathbf{k}) \cdot \left(\frac{3}{2}\cos\left(\frac{t}{2}\right)\mathbf{i} + \frac{3}{2}\sin\left(\frac{t}{2}\right)\mathbf{j}\right) = 0$		
	$\frac{9}{2}\cos\left(\frac{t}{2}\right) - \frac{3}{2}\sin\left(\frac{t}{2}\right) = 0$	m1	
	$\tan\left(\frac{t}{2}\right) = 3$		
	$t = 2\cdot4(9809\ldots) \quad (\text{s})$	A1	cao
		[7]	
	Total for Question 2	11	

Q3	Solution	Mark	Notes
(a)			$\sin\theta = \frac{5}{13}$ $\cos\theta = \frac{12}{13}$ 
	Resolve vertically $R\sin\theta = Mg$	M1 A1	Dimensionally correct
	$\sin\theta = \frac{5}{13}$	B1	si
	$R = Mg \times \frac{13}{5}$		
	$R = \frac{13Mg}{5}$	A1	Convincing
		[4]	
(b)	N2L towards centre $R\cos\theta = Ma$	M1 A1	Dimensionally correct
	$\frac{13Mg}{5} \times \frac{12}{13} = M\frac{(3\sqrt{g})^2}{r}$	m1	$a = \frac{v^2}{r}$
	$CP = r = \frac{15}{4} \quad (= 3\cdot 75 \text{ m})$	A1	
	$\frac{r}{CV} = \frac{5}{12} \quad (= \tan\theta)$	M1	oe, similar triangles
	$CV = 9 \quad \text{(m)}$	A1	cao
		[6]	
	Total for Question 3	**10**	

Q4	Solution	Mark	Notes
(a)	(i) Work-energy principle $16 \times d = 1440\ (000)$	M1	Used, $F \times d = E$
	$d = 90\ (000)$ $\quad$ (km (m))	A1	cao, oe
	(ii) Using expression for PE **or** KE	M1	
	At end, $\text{KE} = \frac{1}{2}(80)(10)^2 \quad = 4000$	A1	$= 4000\ \text{J} = 4\ \text{kJ}$
	$\text{PE} = 80gh = \begin{cases} 11200g = 109760 & \text{for } h = 140 \\ 12000g = 117600 & \text{for } h = 150 \\ 800g = 7840 & \text{for } h = 10 \end{cases}$	A1	
	Work-energy principle (See notes)	M1	All terms included, oe FT their KE and PE
	$WD = 1440\ 000 + 4000 + 109760$	A1	
	$(WD = 1440\ 000 + 4000 + 11200g)$		
	$WD = 1\ 553\ 760$ (J) $\quad$ $(= 1553 \cdot 760\ \text{kJ})$	A1	cao
		[8]	
(b)			$\sin\alpha = \frac{2}{7}$
	$F = \frac{250}{v}$ $\quad$ (maximum force)	B1	si
	Using N2L up plane with $a = 0$ $F - R - mg\sin\alpha = 0$	M1 A1	All forces, dim. correct Correct equation
	$F - 16 - 80g\left(\frac{2}{7}\right) = 0$ $\quad$ $F - 240 = 0$	A1	$\frac{250}{v} - 16 - 80g\left(\frac{2}{7}\right) = 0$
	$v = \frac{25}{24} = 1 \cdot 04(166\ldots)$ $\quad$ (ms^{-1}) (max. speed)	A1	cao
		[5]	
	Total for Question 4	**13**	

	Further Notes
(a)(ii)	Alternative Solution(s) for Work-energy principle • Energy at Start (PE) + WD by cyclist = WD against resistances + Energy at end (PE+KE) $7840 + \text{WD} = 1440\ 000 + 117600 + 4000$ $(800g + \text{WD} = 1440\ 000 + 12000g + 4000)$ • WD by cyclist = WD against resistances + Energy at end (gain in PE+KE) $\text{WD} = 1440\ 000 + 109760 + 4000$ $(\text{WD} = 1440\ 000 + 11200g + 4000)$

Q5	Solution	Mark	Notes
(a)	Impulse $= m_B v_B - m_B u_B$ (change in momentum) $= 2(-6-10)$	M1	Used
	$\lvert\text{Impulse}\rvert = 32 \quad (\text{Ns})$	A1	
	Opposite direction to original motion	A1	
		[3]	
(b)			
	Conservation of momentum	M1	Attempted
	$(4)(3) + (-6)(2) = 3v_A + 2v_B$	A1	All correct
	$3v_A + 2v_B = 0 \qquad \left(v_A = -\frac{2}{3}v_B\right)$		
	Loss in Kinetic energy	M1	Attempted.
	$\frac{1}{2}(3)(4)^2 + \frac{1}{2}(2)(-6)^2 - \frac{1}{2}(3){v_A}^2 - \frac{1}{2}(2){v_B}^2 = 45$	A1	Before $= 60$
	$60 - \frac{3}{2}{v_A}^2 - {v_B}^2 = 45$		After $= \frac{3}{2}{v_A}^2 + {v_B}^2$
	$3{v_A}^2 + 2{v_B}^2 = 30$		
	$3\left(\frac{-2v_B}{3}\right)^2 + 2{v_B}^2 = 30$	m1	One variable eliminated.
	$v_B = \pm 3$ or $v_A = \pm 2$		
	$v_A = -\frac{2}{3}(\pm 3) = \mp 2 \qquad v_B = -\frac{3}{2}(\pm 2) = \mp 3$		
	$(v_A, v_B) = (\mathbf{2}, \mathbf{-3})$ or $(\mathbf{-2}, \mathbf{3})$	m1	
	speed $v_A = 2 \quad (\text{ms}^{-1})$ speed $v_B = 3 \quad (\text{ms}^{-1})$	A1	cao
	Both objects are moving in the opposite direction to their original motion.	A1	
		[8]	
	Total for Question 5	**11**	

Q6	Solution	Mark	Notes
(a)			
	(i) Conservation of energy ($PE = 0$ along horizontal through O) $mg\big(5+7(1-\cos\alpha)\big)=\frac{1}{2}mv^2+mg(5\cos\theta)$ $5\cdot 7g = 0\cdot 5v^2 + 5g\cos\theta$ $55\cdot 86 = 0\cdot 5v^2 + 49\cos\theta$ $2793 = 25v^2 + 2450\cos\theta$ $285g = 25v^2 + 250g\cos\theta$ $v^2 = 111\cdot 72 - 98\cos\theta$ $(= 11\cdot 4g - 10g\cos\theta)$	M1 A1 A1 A1 **[4]**	KE and PE in dim. correct equation KE PE Convincing
	Alternative solution ($PE = 0$ along horizontal through B) (i) Conservation of energy $mg\big(7(1-\cos\alpha)\big)=\frac{1}{2}mv^2-mg\big(5(1-\cos\theta)\big)$ $0\cdot 7g = 0\cdot 5v^2 - 5g + 5g\cos\theta$ $6\cdot 86 = 0\cdot 5v^2 - 49 + 49\cos\theta$ $343 = 25v^2 - 2450 + 2450\cos\theta$ $v^2 = 111\cdot 72 - 98\cos\theta$ $(= 11\cdot 4g - 10g\cos\theta)$	 (M1) (A1) (A1) (A1) **([4])**	 KE and PE in dim. correct equation KE PE Convincing
	(ii) N2L towards O $50g\cos\theta - R = \dfrac{50v^2}{5}$ $R = 50g\cos\theta - \frac{50}{5}(111\cdot 72 - 98\cos\theta)$ $R = \begin{cases} 1470\cos\theta - 1117\cdot 2 \\ 150g\cos\theta - 114g \end{cases}$	M1 A1 m1 A1 **[4]**	Dim. correct equation $50g\cos\theta$, R opposing Substitute for v^2 (any form)

	(iii) Loses contact when $R = 0$ $1470\cos\theta - 1117\cdot 2 = 0$ $\cos\theta = \frac{114}{150}$ $\left(= \frac{1117\cdot 2}{1470} = \frac{19}{25}\right)$ $\theta = 40\cdot 5(358\ldots)^{\circ}$ $\theta = 40\cdot 5(358\ldots)^{\circ} < 45^{\circ}$ $\therefore$ ring loses contact before reaching C	M1 A1 A1 **[3]**	Used. FT R from (ii) or $150g\cos\theta - 114g = 0$ Accept 41 FT R from (ii) FT R provided $\theta < 45^{\circ}$
	Alternative Solution (iii) Sub. $\theta = 45^{\circ}$ into expression for R $R = -77.7(530\ldots)$ $R = -77.7(530\ldots) < 0$ $\therefore$ ring loses contact before reaching C	 (M1) (A1) (A1) **([3])**	 FT R from (ii) FT R FT provided $R < 0$
	(iv) Rubber ring may lose contact at a greater value of θ or Rubber ring may remain in contact until C.	E1 **[1]**	
(b)	N2L towards D At A, $\cos\alpha = 0\cdot 9$: $R_A - 50g(0\cdot 9) = 50a$ $R_A = 50a + 45g > 45g = 441$ At B, $\cos\alpha = 1$: $R_B - 50g = 50a$ $R_B = 50a + 50g > 50g = 490$ $R_B > R_A > 0$ so must remain in contact	M1 A1 A1 **[3]**	Dim. correct equation Any correct equation including $R' - 50g\cos\alpha = 50a$ $a = \frac{v^2}{r} > 0$ Convincing
	Total for Question 6	**15**	